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Estimating Liquidity Augmented Three Factor Model for PSX

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ABSTRACT

Objectives: Several risk factors have been identified as crucial determinants of cross-sectional variations in assets' returns in three, four, and five-factor models. This paper aims incorporates liquidity, in two separate measures with volume and volume as percentage of outstanding shares, in standard three factor model by considering hundred companies from Pakistan Stock Exchange (PSX).

Research Gap: Previous relevant literature for PSX considers other factors but could not found for estimating three factor model with liquidity factor. However, these types of models are developed for other markets.

Methodology: It takes data of 100 selected companies of PSX to estimate standard three factor and liquidity-augmented three factor models. Portfolio construction and estimations are carried out as per the methodology developed by Fama and French in factor models.

The Main Findings: Based on the estimations of these liquidity augmented three factor models, it is concluded that investors in PSX require premium against investment in illiquid portfolios along with the other risk factors such as market, size, and value.

Practical implications of findings: The better choice for risk-avert rational investors is to construct a portfolio with big size, low book to market value, and liquid assets' companies.

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1. Introduction

Stock exchanges provide opportunities to companies to obtain equity while it hinges upon the investors required rate of return which depends upon going risk free rate and risk factors. More risks lead to higher required rate of return thus lower stock prices. Hence, study and implementation of asset pricing tools remains a popular research theme amongst financial econometricians and practitioners. It leads us to ascertain price of an asset subject to one or more risk factors. Thus, asset pricing models may also be utilized for forecasting price and spotting investment opportunities.

The relationship between risk and return studied by Markowitz (1952), the theory of asset pricing came into limelight with the inception of Capital Asset Pricing Model (CAPM) by Sharpe (1964) and Lintner (1965). Some other popular asset pricing models include Intertemporal Capital Asset Pricing Model of Merton (1973),

Arbitrage Pricing theory (APT) of Ross (1976) and Fama and French (FF) three factor model (1990, 1993), which is an extension of conventional CAPM. Fama and French (1996) showed that their three-factor form is able to capture 96% of the variations in equities listed in New York Stock Exchange (NYSE). Despite higher explanation power, many researchers have extended the work of Fama and French (1990, 1992) by incorporating new factors including Fama and French (2015).

With several international markets, the asset pricing model are also developed for Pakistan Stock Exchange (PSX). In most of the studies pertaining to PSX, asset returns are ascertained through CAPM. However, a sizeable amount of literature in the context of PSX also applied three factors and five factors model. This paper uses the three-factor model with an additional liquidity factor, making it liquidity-augmented three factor model.

The urge of using liquidity stems from the fact that low liquidity is itself a risk and rational investor should require more return as the risk increases. Trading volume for a firm can be taken as a measure of its liquidity (Brennan, Chordia, and Subrahmanyam, 1998). Higher trading volume suggests higher liquidity. One may argue that the trading volume may also depend upon the number of outstanding shares. A company which has large number of outstanding shares may also have higher trading volume and vice versa. Therefore, another measure of liquidity may be trading volume by outstanding shares, also known as turnover (Amihud and Mendelson 1986).

This paper is conceived with an intent to augment the standard three factor model for PSX with another factor of liquidity with its two measures such as average trading volume and trading volume / outstanding shares. Previously, these measures are not used with three factor models for PSX. These measures are used earlier in the studies for other stock markets such as Rahim and Nor (2006), Amihud (2002), Amanda and Husodo (2015). In the context of the PSX, which is a comparatively smaller market than that of the global stock exchanges, the liquidity factor is more concerning to the investors as smaller number of investors / traders make several stocks dry and difficult to liquidate at the given market price. Investors / traders have to bear with this risk and they may require premium against it. This problem, as not explicitly empirically investigated in the earlier literature for the PSX, requires research and understanding of the importance of liquidity risk for the investors / traders in the market, which is contributed by this paper with the following objectives.

1.1 Objectives of the Research

Objectives of this paper are

a. Estimation of the standard three-factor model for selected PSX stocks
b. To estimate the liquidity augmented three factor models, also referred to as four factor models, with volume (model-1) and volume / outstanding shares (model-2)

This paper helps understand the liquidity as an important determinant for explaining the cross-sectional variations in assets' returns. Further, the estimations of three and four factor models also illustrate the behavior towards risk in PSX. Findings of this paper can help investors to construct portfolios by incorporating relevant risk factors.

Rest of the paper consists of four sections. Literature review of comparable studies is presented in next section followed by methodology for the portfolio formation and empirical estimations. The fourth section comprises of empirical results obtained from estimation of models. The last section presents conclusions and recommendations.

2. Review of Literature

This section presents review of relevant studies which have used FF model in the context of PSX. These studies include implementation of FF model and identifying a superior portfolio. It also includes literature that

compares the explanatory power of FF models with the CAPM.

Abbas et al. (2015) tested FF three factor for Pakistani equity market via individually observing the impact of market, size, and value premiums on equity portfolio return for time period of 2004 – 2014. These portfolios are constructed by the intersection of two factors – market capitalization and book to market equity. The study found that market premium, distress and leverage largely hold valid for the securities listed in Pakistani equity market. Despite standing above 62% for all the six portfolios, the coefficient of determination was comparatively low to original results of FF which signifies that predictability of the model is relatively low in case of emerging equity markets.

Shoaib and Siddiqui (2016) applied FF model for PSX using a Panel approach. In this study the FF test was applied on annual basis for all companies listed in PSX during 2001 to 2010. The study held FF model valid for all investigated years. Butt and Sadaqat (2019) applied FF model to investigate that weather the returns while investing into Shariah Compliant stocks are reduced as compared to the conventional stocks. They have carried out screening of all stocks listed in PSX during 2004 – 2016 through the methodology of Ince and Porter (2006) and Griffin et al. (2010). Upon screening they found 85 and 112 Shariah compliant and conventional stocks respectively for the implementation of FF model. Then they have formed eight portfolios for both categories of stocks and concluded that earnings on Shariah compliant stocks are not substantially lower than the conventional stocks.

Hassan and Javed (2011) compared the CAPM and three factor FF model for 250 companies listed at PSX during FY01 to FY07 and found that adjusted R-square values for the FF three portfolios on average stayed above 15% for FF model as compared to CAPM. Latif et al. (2014) also compared the validity of CAPM and FF model for PSX via its implementation on securities pertaining to 21 out of 35 sectors for the period of January 2001 to December 2009. Fama and French model was applied in its typical six portfolio fashion. The results showed superiority of FF model over CAPM, where within six portfolios, S/M exhibited highest predictability. The superiority of FF over CAPM was also established in the work of Iqbal et al. (2017) on a restricted portion of PSX (i.e. 40 companies) for the time period of 1984 to 2012 through Panel least square estimation. Another restricted study, pertaining to exclusive of some 20 banks was carried out by Hamid et al (2012). The time period considered in Hamid et al. spans from January 2006 to December 2010. The study found that intercept values of all standard FF portfolios are insignificant, the R-square values ranged between 43% and 70%, which are comparatively lower than the portfolios of developed economies.

Latif et al. (2014) and Iqbal et al. (2017), Mirza and Reddy (2017) also compared the predictability of excess returns through CAPM, conventional three factor FF model and its four-factor extension on PSX. The fourth factor which was incorporated in their model was momentum as described by Carhart (1997). The time period considered in this study was FY90 to FY15. Using the technique of Liew and Vassalou (2000), they formed 12 portfolios for testing FF three and four factor models. The results revealed that FF four factor model outperform other two models considered in this study, however, the explanatory power of portfolios remained below 40%.

Ghani et al. (2022) extended the CAPM model by incorporating liquidity with systematic risk. Measuring liquidity with market volume it concluded impact of liquidity on risk premiums especially on highly liquid stocks. Khan et al. (2022) uses zero return as a measure of illiquidity and incorporate it with other risk factors and control variables. With a comparatively smaller size of data (thirty companies from Pakistan Stock Exchange and London Stock Exchange each), it concludes insignificant impact of illiquidity on stock returns. Ahmed et al. (2023) also worked on liquidity by employing liquidity ratios such as return to volume and return to turnover ratios. It concluded that the return to turnover is more effective and liquidity is a significant factor in determining stock returns, based on the estimations of alphas.

Despite the superiority of FF models in explaining returns for PSX, it is largely found that predictability of FF

model in the context of PSX has exhibited limited power. Thus, incorporation of liquidity factor may enhance predictability of FF model for emerging markets. Limited literature about liquidity risk exists in previous research with factor modelling and lacks the risk premium estimation associated with it.

3. Methodology

Methodology used in this paper comprises of data formation, construction of portfolios and econometric models. This research is explanatory and mainly concerned with the impact and importance of three factors market premium, size, and value along with the fourth factor of liquidity measured with volume and volume / outstanding shares.

3.1 Data

Total number of listed companies in PSX is five hundred and twenty-five. Daily trading data of all those companies, for the period of July 2009 to June 2020, is taken from the PSX. Data for the years 2021 onwards is not included to save the analysis from extraordinary variations in Covid period. 100 companies are finally selected having less than 5% missing values as large number of missing values contaminates results. It also saved the analysis from the bias of dry market and extraordinary illiquid stocks. Similarly, the daily volume and outstanding shares of all the 100 stocks is taken from the PSX to construct two measures of liquidity, volume and volume / outstanding shares. Further the KSE-100 index data is taken as the representative index.

Financial information of the companies is taken from the State Bank of Pakistan (SBP). It includes data of ordinary share capital used to compute the number of shares by dividing it with par value. This data is taken for the period of July 2008 to June 2019. Values are taken as of June of each year. Further, the data of 3-months T-bills is taken from the SBP. The annualized T-bill values are divided by number of trading days to convert them to the daily risk-free rate.

3.2 Construction of Portfolios

Three models are estimated in this paper. One is the standard three factor model and two are the liquidity augmented three factor models with volume and volume / outstanding shares separately. Variables constructed for the three models are discussed below.

3.3 Three Factor Model

The standard three factor model includes market premium, size, and value factors. The size is measured by the market equity, price times number of outstanding shares. Value is measured by the book to market equity, shareholders' value to the market equity. It constructs six portfolios based on the size and value. First it sorts the companies by size and considers the top 50 percent as big and bottom 50 percent as small. Then it sorts the companies by book to market value and takes top 30 percent companies as high value, bottom 30 percent as low value and middle 40 percent as medium value companies. Intersection of these sorts of size and value makes six portfolios like big size and high value (BH), big size and medium value (BM), big size and low value (BL), small size and high value (SH), small size and medium value (SM) and small size and low value (SL). This process of portfolio construction is repeated at the last day of June for the year t. It uses information available at time t to construct portfolio for time t+1. All the six portfolios are weighted by size. The formula is as follows:

$$\text{Portfolio Return} = \frac{\sum_{i=1}^{n} r_i w_i}{\sum_{i=1}^{n} w_i} \quad (1)$$

After constructing these portfolios, the factors for size and value effects are generated. The size factor SMB is generated by taking difference of average returns of small firms' portfolios (SH, SM, SL) and average returns of big firms' portfolios (BH, BM, BL). Similarly, the value factor HML is generated by taking difference of average returns of high value firms' portfolios (BH, SH) and average returns of low value firms' portfolios (BL, SL).

Table 1: Construction of Portfolios and Risk Factors

Sort	Breakpoints	Factors and their components
Three Factor Model 2 x 3 sort on size and book to market (B/M) value	Size: median size of all companies B/M: 30th and 70th percentile	$SMB = \frac{SH + SM + SL}{3} - \frac{BH + BM + BL}{3}$ $HML = \frac{SH + BH}{2} - \frac{SL + BL}{2}$
Four Factor Model – 1 2 x 3 x 2 sort on size, book to market (B/M) value, and volume	Size: median size of all companies B/M: 30th and 70th percentile Volume: median volume of all companies	$SMB = \frac{SHQ + SMQ + SLQ + SHN + SMN + SLN}{6} - \frac{BHQ + BMQ + BLQ + BHN + BMN + BLN}{6}$ $HML = \frac{BHQ + BHN + SHQ + SHN}{4} - \frac{BLQ + BLN + SLQ + SLN}{4}$ $NMQ = \frac{BHN + BMN + BLN + SHN + SMN + SLN}{6} - \frac{BHQ + BMQ + BLQ + SHQ + SMQ + SLQ}{6}$
Four Factor Model – 2 2 x 3 x 2 sort on size, book to market (B/M) value, and volume/outstanding shares	Size: median size of all companies B/M: 30th and 70th percentile Volume/outstanding shares: median value of volume / outstanding shares of all companies	$SMB = \frac{SHQ + SMQ + SLQ + SHN + SMN + SLN}{6} - \frac{BHQ + BMQ + BLQ + BHN + BMN + BLN}{6}$ $HML = \frac{BHQ + BHN + SHQ + SHN}{4} - \frac{BLQ + BLN + SLQ + SLN}{4}$ $NMQ = \frac{BHN + BMN + BLN + SHN + SMN + SLN}{6} - \frac{BHQ + BMQ + BLQ + SHQ + SMQ + SLQ}{6}$

Source: Authors' compilation

3.4 Four Factor Model – 1

The model – 1 is an extension of the standard three factor model, the three factors include market premium, size, and value factors. The fourth factor in model-1 is volume. The companies are sorted by average daily volume generated in a year and then by volume. Top 50 percent companies are taken as liquid companies shown by 'Q' and bottom 50% are less liquid shown by 'N'. Intersection of sorted companies by size, value, and volume makes twelve portfolios like big size, high value, and liquid companies (BHQ), big size, high value, and illiquid companies (BHN), big size, medium value, and liquid companies (BMQ), big size, medium value, and illiquid companies (BMN), big size, low value, and liquid companies (BLQ), big size, low value, and illiquid companies (BLN), small size, high value, and liquid companies (SHQ), small size, high value, and illiquid companies (SHN), small size, medium value, and liquid companies (SMQ), small size, medium value, and illiquid companies (SMN), small size, low value, and liquid companies (SLQ), and small size, low value, and illiquid companies (SLN). This process of portfolio construction is repeated at the last day of June for the year t. It uses information available at time t to construct portfolio for time t+1. All the twelve portfolios are weighted by size.

After constructing portfolios, the factors for size, value, and liquidity effects are generated. The size factor SMB is generated by taking difference in average returns of small firms' portfolios (SHQ, SHN, SMQ, SMN, SLQ, and SLN) and average returns of big firms' portfolios (BHQ, BHN, BMQ, BMN, BLQ, and BLN). The value factor HML is generated by taking difference in average returns of high value firms' portfolios (BHQ, BHN, SHQ, and SHN) and average returns of low value firms' portfolios (BLQ, BLN, SLQ, and SLN). The liquidity factor NMQ is generated by taking difference in average returns of illiquid firms' portfolios (BHN, BMN, BLN, SHN, SMN, SLN) and average returns of liquid firms' portfolios (BHQ, BMQ, BLQ, SHQ, SMQ, SLQ).

3.5 Four Factor Model – 2

The model – 2 is another extension of the standard three factor model. The fourth factor in model-2 is average volume / outstanding shares along with the three factors. The companies are sorted by average volume / outstanding shares generated in a year. The companies are then sorted by volume / outstanding shares. Top 50 percent companies are taken as liquid companies shown by 'Q' and bottom 50% are less liquid shown by 'N'. The intersection of sorted companies by size, value, and volume / outstanding shares makes twelve portfolios

similar to the model-1. The size, value, and liquidity factors are also developed as same as in the model-1. Construction of portfolios with formulas are shown in Table-1.

3.6 Model Used for Estimations

The model used in the estimation is ordinary least square. It includes six regressions for three factor model and twelve regressions for each of the two four-factor models. Dependent variables are risk premium on constructed portfolios and independent variables are market risk premium and other risk factors as constructed in three factor model based on size and value and in four factor models such as size, value, and liquidity. The estimation models are as follows

For Three Factor Model

$$R_{it} - R_{Ft} = \alpha_0 + \alpha_1(R_{Mt} - R_{Ft}) + \alpha_2 SMB_t + \alpha_3 HML_t + \varepsilon_t \quad (2)$$

For Four Factor Models

$$R_{it} - R_{Ft} = \beta_0 + \beta_1(R_{Mt} - R_{Ft}) + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 NMQ_t + \varepsilon_t \quad (3)$$

Where, 'i' represents portfolio 'i'. R_{it} shows return on portfolio 'i' for time period 't', R_{Ft} represents return on risk free asset, treasury security, for time period 't', R_{Mt} indicates return on market portfolio, KSE-100 index, for time period 't', SMB_t, HML_t, NMQ_t are size, value, and liquidity factors for time period 't'.

4. Estimations

Mean and standard deviations of risk premium and risk factors for the three and four-factor models are shown in Table 2. Majority of portfolios' risk premiums are positive showing that taking risk in the market gives more return as compared to risk free rate on average. Risk premium on market as shown by KSE, which is KSE-100 index return minus risk-free rate, is also positive. It shows that average return on small companies' portfolios is higher than big companies' portfolios as shown by positive value of SMB in the three models. Similarly, the average return on high value companies' portfolios is higher than the low value companies' portfolios as represented by positive value of HML in the three models. The fourth factor of liquidity as shown by NMQ in the Four-Factor Model – 1 and Four-Factor Model – 2 are also showing positive mean values indicating that the average return on illiquid portfolios is higher than the average return on liquid portfolios. In addition to this, it also shows that both the measures of liquidity, volume and volume / outstanding shares, are consistent in showing liquidity as an important factor for determining cross sectional variations in return on assets. The consistency found in the mean values of SMB, HML, and NMQ is not evident in the standard deviations of HML and NMQ but in SMB.

Table 2: Mean and Standard Deviation of Risk Premiums and Risk Factors by Portfolios

Three Factor Model			Four Factor Model – 1 (Volume)			Four Factor Model – 2 (Volume / Outstanding Share)		
Portfolio	**Mean**	**St. dev**	**Portfolio**	**Mean**	**St. dev**	**Portfolio**	**Mean**	**St. dev**
SH	0.00023	0.01633	**SHQ**	0.00001	0.02010	**SHQ**	0.00009	0.02001
SM	-0.00016	0.01336	**SHN**	0.00055	0.01527	**SHN**	0.00042	0.01555
SL	0.00031	0.01929	**SMQ**	-0.00056	0.02065	**SMQ**	-0.00051	0.01743
BH	0.00015	0.01530	**SMN**	0.00012	0.01324	**SMN**	0.00011	0.01320
BM	-0.00005	0.01271	**SLQ**	0.00044	0.02881	**SLQ**	0.00050	0.02301
BL	-0.00011	0.01144	**SLN**	0.00055	0.02884	**SLN**	0.00025	0.04428
KSE	0.00020	0.01065	**BHQ**	0.00010	0.01654	**BHQ**	0.00015	0.01912
SMB	0.00013	0.00839	**BHN**	0.00027	0.01873	**BHN**	0.00017	0.01666
HML	0.00008	0.00919	**BMQ**	-0.00008	0.01354	**BMQ**	-0.00017	0.01507
			BMN	-0.00001	0.01272	**BMN**	0.00002	0.01279
			BLQ	-0.00016	0.01327	**BLQ**	0.00022	0.01503
			BLN	-0.00020	0.01115	**BLN**	-0.00030	0.01123

KSE	0.00020	0.01065	**KSE**	0.00020	0.01065
SMB	0.00020	0.00984	**SMB**	0.00015	0.01019
HML	0.00008	0.01051	**HML**	0.00003	0.01263
NMQ	0.00026	0.00918	**NMQ**	0.00007	0.01048

Source: Authors' calculations

Estimations results of the Three-Factor model as shown in equation (2) are presented in the Table 3. The results suggest that all portfolios premium have positive relationship with the market risk premium. Two of the portfolios, SM and BL, are found less risky than market whereas the others are riskier than the market. The factors of size and value are also significantly affecting the portfolios' risk premium.

Interpreting the cross-sectional variations in assets' return due to size, it analyzes if the small portfolios' coefficients of $\boldsymbol{SMB_t}$ $(\boldsymbol{\alpha_2})$ is higher than the big portfolios' coefficients of $\boldsymbol{SMB_t}$ $(\boldsymbol{\alpha_2})$ keeping the other factors constant. Similarly, for value factor, it considers if the high value portfolios' coefficients of $\boldsymbol{HML_t}$ $(\boldsymbol{\alpha_3})$, is higher than the low value portfolios' coefficients of $\boldsymbol{HML_t}$ $(\boldsymbol{\alpha_3})$, keeping the other factors constant. Table 4 highlights difference in the coefficients of $\boldsymbol{SMB_t}$ and $\boldsymbol{HML_t}$.

Table 3: Estimation Results of Three Factor Model

Portfolio $R_{it} - R_{Ft}$	Constant	KSE-100 $R_{Mt} - R_{Ft}$	SMB_t	HML_t	R-sq.	Adj. R-sq.	F-statistic
	α_0	α_1	α_2	α_3			
SH	-0.00019*	1.05858***	1.04070***	0.72731***	0.90	0.90	8365.7***
SM	-0.00045***	0.92168***	0.67194***	0.22732***	0.74	0.74	2597.8***
SL	-0.00005	1.16339***	1.29265***	-0.52222***	0.82	0.82	4110.6***
BH	-0.00015	1.09424***	0.16630***	0.56657***	0.75	0.75	2659.2***
BM	-0.00026**	1.06000***	-0.07533***	0.04971***	0.80	0.80	3585.8***
BL	-0.00028***	0.98943***	-0.08568***	-0.18391***	0.84	0.84	4702.2***

Source: Authors' estimations

The first value in SMB factor, which shows difference in the coefficients associated with size factor of SH and BH portfolios, is 0.874. It indicates that size factor affects SH portfolio higher than BH portfolio by 0.874 units. A similar interpretation can be given for difference in SM, BM and SL, BL. For value factor, the first value is 0.500 which suggests that the value factor affects SH portfolio higher than SM portfolio by 0.500. A similar interpretation can be given for difference in SM, SL and BH, BM and BM, BL.

Table 4: Difference in Size and Value Coefficients of Three Factor Model

Difference in portfolios	Difference in the coefficients of Three Factor Model
SMB Factor	
SH – BH	0.874
SM – BM	0.747
SL – BL	1.378
HML Factor	
SH – SM	0.500
SM – SL	0.750
BH – BM	0.517
BM – BL	0.234

Source: Authors' estimations

Estimations of the four-factor models include liquidity as the fourth factor in addition to the three factors as incorporated in the three-factor model with volume and volume / outstanding shares as Four-Factor Model-1 and Four-Factor Model-2 respectively. Estimation of these two models as per equation (3) are presented in the Table-5 and Table-6.

The model-1, with volume as liquidity factor shows, in Table-5, that all portfolios' risk premiums have positive relationship with market risk premium. However, the quantum of this relationship varies as the coefficient values ranges from 0.85 to 1.32 showing that some portfolios are less risky than the market while the other are risker than the market. A similar pattern is also observed in the estimations of model-2, with volume / outstanding shares. The relationship between portfolios' risk premiums and market risk premium is statistically significant across all portfolios, while the effect ranges from 0.78 to 1.53 showing presence of less risky and risker than market portfolios.

Table 5: Estimation Results of Four Factor Model-1 (Volume as Liquidity Factor)

Portfolio Risk Premium $R_{it} - R_{Ft}$	Constant β_0	KSE-100 $R_{Mt} - R_{Ft}$ β_1	SMB_t β_2	HML_t β_3	NMQ_t β_4	R-sq.	Adj. R-sq.	F-statistic
SHQ	-0.00042 **	1.00335 ***	1.23913 ***	0.75943 ***	-0.30470 ***	0.81	0.81	2870.0 ***
SHN	0.00003	0.95406 ***	0.92021 ***	0.68626 ***	0.36557 ***	0.72	0.72	1738.2 ***
SMQ	-0.00085 ***	1.03713 ***	0.86928 ***	0.35835 ***	-0.45060 ***	0.67	0.67	1344.2 ***
SMN	-0.00026	0.86530 ***	0.55596 ***	0.24452 ***	0.30369 ***	0.58	0.57	910.1 ***
SLQ	0.00039	0.91700 ***	1.07183 ***	-0.47603 ***	-1.24621 ***	0.72	0.72	1761.2 ***
SLN	-0.00042	1.32331 ***	1.61539 ***	-0.77368 ***	1.70888 ***	0.79	0.79	2509.7 ***
BHQ	-0.00012	1.04947 ***	0.29702 ***	0.35100 ***	-0.31544 ***	0.69	0.69	1470.6 ***
BHN	-0.00021	1.12187 ***	0.11450 ***	0.67197 ***	0.67072 ***	0.43	0.43	512.6 ***
BMQ	-0.00025 **	1.06129 ***	-0.06241 ***	-0.02254	-0.13950 ***	0.77	0.77	2189.5 ***
BMN	-0.00028 *	0.97901 ***	0.03899 **	0.08003 ***	0.20733 ***	0.59	0.59	964.6 ***
BLQ	-0.00029 **	1.03189 ***	-0.14409 ***	-0.17140 ***	-0.16690 ***	0.77	0.77	2284.7 ***
BLN	-0.00040 ***	0.85658 ***	0.02670 *	-0.11026 ***	0.12038 ***	0.63	0.63	1148.9 ***

*** significance at 1%, ** significance at 5%, * significance at 10%

Source: Author's estimations

In the two models, to interpret the cross-sectional variations in the assets' return due to size, difference in the small and big portfolios' coefficients of SMB_t (β_2) keeping the other factors constant are crucial. For value factor, difference between the high-value and medium-value portfolios' coefficients of HML_t (β_3) and medium-value and the low-value portfolios' coefficients of HML_t (β_3) are computed keeping the other factors constant. Similarly, for value factor, difference in the coefficients of NMQ_t (β_4) of illiquid and liquid companies' portfolios, keeping the other factors constant, are calculated. Table 7 highlights difference in the coefficients of SMB_t, HML_t, and NMQ_t.

Table 6: Estimation Results of Four Factor Model-2 (Volume/Outstanding Shares as Liquidity Factor)

Portfolio Risk Premium $R_{it} - R_{Ft}$	Constant β_0	KSE-100 $R_{Mt} - R_{Ft}$ β_1	SMB_t β_2	HML_t β_3	NMQ_t β_4	R-sq.	Adj. R-sq.	F-statistic
SHQ	-0.00030 *	1.14177 ***	1.21821 ***	0.56896 ***	-0.49679 ***	0.82	0.82	3065.0 ***
SHN	0.00005	0.97404 ***	0.90706 ***	0.76952 ***	0.30470 ***	0.70	0.70	1564.8 ***
SMQ	-0.00078 ***	0.98294 ***	0.73921 ***	0.15621 ***	-0.59835 ***	0.73	0.73	1855.5 ***
SMN	-0.00013	0.77796 ***	0.50506 ***	0.27768 ***	0.05594 ***	0.49	0.49	644.6 ***
SLQ	0.00025	0.95353 ***	1.03261 ***	-0.28774 ***	-1.18816 ***	0.73	0.73	1788.0 ***
SLN	-0.00037	1.52611 ***	1.93727 ***	-1.11931 ***	1.46421 ***	0.86	0.86	3639.4 ***
BHQ	-0.00011	1.14816 ***	0.38552 ***	0.28806 ***	-0.48462 ***	0.66	0.66	1321.2 ***
BHN	-0.00014	1.12817 ***	0.18565 ***	0.52380 ***	0.52440 ***	0.50	0.50	679.4 ***
BMQ	-0.00039 ***	1.11075 ***	0.07670 ***	-0.02369	-0.27588 ***	0.75	0.75	2063.6 ***
BMN	-0.00019	1.04671 ***	-0.10480 ***	0.01425	0.15966 ***	0.70	0.70	1555.6 ***
BLQ	0.00007	1.01813 ***	-0.14411 ***	-0.32109 ***	-0.41781 ***	0.74	0.73	1864.4 ***
BLN	-0.00047 ***	0.90234 ***	-0.07309 ***	-0.09789 ***	0.02033	0.72	0.72	1751.5 ***

*** significance at 1%, ** significance at 5%, * significance at 10%

Source: Author's estimations

In addition to the market risk premium, other factors include size, value, and liquidity. The size factor, SMB,

shows statistically significant relationship across all portfolios. However, to understand the impact of size, difference in the coefficients of SMB across different small and big companies' portfolios are calculated, as shown in Table-7. All differences in coefficients are found positive reflecting impact of size on companies' return. For instance, the first difference between small size, high value, and liquid companies' portfolio, SHQ, coefficient of SMB is higher than big size, high value, and liquid companies' portfolio, BHQ, coefficient of SMB by 0.942 units. Similar interpretation can be given for other difference in coefficients as well. It can be concluded that similar to Three-Factor Model, Four-Factor Model-1 also confirms the impact of size on companies' return. Likewise, the value factor, HML, also shows statistically significant relationship across all portfolios except for BMQ portfolio. To understand the impact of value factor, difference in coefficients of HML across multiple high and medium value portfolios and medium and low value portfolios is mentioned in Table-7 for Model-1. For example, the first value, 0.401, shows that small size, high value, and liquid companies' portfolio, SHQ, coefficient of HML is higher than small size, medium value, and liquid companies' portfolio, SMQ, coefficient of HML by 0.401 units. All differences are found positive showing that the value factor affects companies return. High value companies have higher return compared to medium value companies and medium value companies have higher return than low value companies.

Table 7: Difference in Size, Value, and Liquidity Coefficients of Four Factor Model-1 and Model-2

Difference in portfolios	Four Factor Model-1	Four Factor Model-2
SMB Factor		
SHQ – BHQ	0.942	0.833
SHN – BHN	0.806	0.721
SMQ – BMQ	0.932	0.663
SMN – BMN	0.517	0.610
SLQ – BLQ	1.216	1.177
SLN – BLN	1.589	2.010
HML Factor		
SHQ – SMQ	0.401	0.413
SMQ – SLQ	0.834	0.444
SHN – SMN	0.442	0.492
SMN – SLN	1.018	1.397
BHQ – BMQ	0.351	0.288
BMQ – BLQ	0.171	0.321
BHN – BMN	0.592	0.524
BMN – BLN	0.190	0.098
NMQ Factor		
SHN – SHQ	0.670	0.801
SMN – SMQ	0.754	0.654
SLN – SLQ	2.955	2.652
BHN – BHQ	0.986	1.009
BMN – BMQ	0.347	0.436
BLN – BLQ	0.287	0.418

Source: Author's estimations

The fourth factor of liquidity in Four-Factor Model-1, which is the contribution of this paper, is shown by NMQ. Coefficients of this factor are found statistically significant across all twelve portfolios. Similar to the difference in coefficients of SMB and HML factors for size and value, difference in coefficients of NMQ, liquidity factor, are also calculated and shown in Table-7. The first value, 0.670, shows that the small size, high value, and illiquid companies' portfolio, SHN, coefficient of NMQ is higher than the small size, high value, and liquid companies' portfolio, SHQ, by 0.670 units. All the other difference in coefficients of NMQ between illiquid and liquid companies' portfolios are positive showing impact of liquidity on companies' return.

Similar to the model-1, the estimation results of model-2, with volume/ outstanding shares as a measure of liquidity factor, also show statistically significant impact of size, value, and liquidity across all portfolios except for the HML coefficient of BMQ and BMN portfolios and for the NMQ coefficients of BLN portfolio. All differences to identify the impact of size, value, and liquidity factors are presented in Table-7 for model-2 as well. Consistent pattern is found for model-2 like model-1 showing positive values of differences across the board. Hence, the size, value, and liquidity factors are important to explain cross-sectional variations in assets' returns.

5. Conclusions and Recommandations

Estimation of the three and four factor models along with an additional factor of liquidity in this paper for the portfolios constructed on the stocks listed in the PSX suggest that size, value, and liquidity are the important determinants of the cross-sectional variations in assets' return. Assets with big size tend to have lower return as compared to assets with small size. Similarly, assets with lower book to market value have lower return compared to assets with higher book to market value. On the liquidity, where the factor is gauged with volume and volume / outstanding shares, it can be concluded that liquidity is another important determinant of cross-sectional variations in assets' return in PSX. Investors require higher return against the liquidity risk. Further, the two measures of liquidity may help understand this phenomenon.

It is also crucial to note that the market and investors in PSX may be treated as rational because they require higher returns against the higher risk stemming from difference in size, value, and liquidity. Further, this approach is found consistent across various portfolios.

Based on the results and conclusion, it is critical to consider size, value, liquidity as important factors while constructing portfolios. An individual investor, fund managers, and wealth managers may evaluate the behavior towards risk and suggest the portfolio accordingly. Return on the portfolios with big size, low book to market value, and / or high liquidity are close to the market return. Hence, a better choice for risk avert rational investors is to construct a portfolio with big size, low book to market value, and liquid assets. Specifically, they should refrain from small size, low value, and illiquid stocks as the estimation suggest that one would need to have comparatively higher premiums on this than the other portfolios. The liquidity risk may be reduced to some extent by encouraging more investments in stocks. The government needs to give incentives in terms of lower capital gain tax as it may result in more investment in market, lowering liquidity risk, and ultimately getting larger collection of capital gain tax.

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